23BMT452

UG PROGRAM (4 YEARS HONORS) WITH SINGLE MAJOR

AT THE END OF FOURTH SEMESTER

MATHEMATICS-INTRODUCTION TO REAL ANALYSIS & PROBLEM SOLVING SESSIONS

(Minor)

(w.e.f. Admitted Batch 2023-24)

Time: 3 Hours **Maximum: 70 Marks**

Section–A

Answer any Five Questions. **5x4=20**

1. Show that $lim \sqrt{\frac{n+1}{n}} = 1$

2. Test for the convergence of $\sum \left(1 + \frac{1}{n}\right)^{-n^2}$

3. If $f(x) = \frac{x^5-2^5}{x-2}, x \neq 2$ is continuous at $x = 2$, then define $f(2)$.

4. Find c of Cauchy's mean value theorem for f (x) = $x^2$and g(x) =x^3 in [1, 2]

5. If $f(x) = k \; \forall x \in [a,b]$ where k is a real number, show that $f \in R[a,b]$

6. Show that every convergent sequence is bounded.

7. State and prove Leibniz test.

8. Prove that the function defined by

   f (x) = $\left[\begin{array}{l} 0 \; when \; x \in Q \\ 1 \; when \; x \in R - Q \end{array} \right.$ is not integrable on any interval of R

Section– B

Answer All Questions. **5x10=50**

9. a) Test for the convergence of 1+ $\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$

(Or)

b) Show that the $\{s_n\}$ defined by $s_1 = \sqrt{2}, \quad s_{n+1} = \sqrt{2s_n}$ converges to 2

10. a) State and prove D'Alembert's Ratio Test.

(Or)

b) Test for the convergence of $\sum \frac{(-1)^n}{n}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}\right)$

11. a) Let f: [a,b]→R. If f continuous on[a,b] and f(a), f(b) have opposite signs then show that there exists $c \in (a,b) \ni f(c) = 0$

(Or)

b) Examine the continuity of the function f defined by $f(x) = |x| + |x - 1|$ at $x = 0$ *and* 1

12. a) Verify Rolle's theorem for the function f (x) = $(x - a)^m (x - b)^n$ *in* [a, b] when m and n being positive integers.

(Or)

b) State and prove Lagrange's mean value theorem.

13. a) If $f : [a, b] \to R$ is continuous on $[a, b]$ then show that f is R – integrable on $[a, b]$

(Or)

b) Show that a bounded function f: [a, b]→ R is Riemann integrable on [a, b] if and only if for each $\varepsilon > 0$ there exists a partition P of [a, b] such that $0 \leq U(P,f) - L(P,f) < \varepsilon$